Chapter 22
THE COST OF EQUITY

A ship in a harbour is safe but is not what ships are built for

The previous chapter described the important concepts of risk, return and market portfolio. It also highlighted the notion of risk premium (i.e. the difference between the risk-free rate and the return on the market portfolio), and this chapter continues to explore the risk premium in greater depth.

By seeking to systematically estimate the risk premium, i.e. in a fairly valued market, the question arises: what risk premium must be added to the risk-free rate to determine the required rate of return?

Investors must look at the big picture, first by investing in the market portfolio, then by borrowing or by investing in risk-free instruments commensurate with the level of risk they wish to assume. This approach allows them to assess an investment by merely determining the additional return and risk it adds to the market portfolio.

Investment risk is often broken down into its component parts, not necessarily in economic and financial terms, but rather into the volatility of the security itself and the volatility of the market as a whole.

We want to know how to get from r (the discounting rate used in calculating company value) to k (the return required by investors on a specific security).

Remember that this approach applies only if the investor owns a perfectly diversified portfolio. Here is why: the greater the risk assumed by the financial investor, the higher his required rate of return. However, if he makes **just one** investment and that turns out to be a failure, his required rate of return will matter little, as he will have lost everything.

With this in mind, it is easier to understand that risk premium is relevant only if the financial investor manages not just a single investment, but a diversified portfolio of investments. In this case, the failure of one investment should be offset by the return achieved by other investments, which should thereby produce a suitable return for the portfolio as a whole.

The concept of risk premium only makes sense when risk is spread over many investments.

Look at it this way:

- What is a bank if not a portfolio of loans and financial resources?
- What is a financial group if not a portfolio of industrial investments?
- What is a mutual fund if not a portfolio of securities?
- What is an insurance company if not a portfolio of statistical risks?

"Portfolio" and "diversification" are fundamental concepts in finance.

This is the main difference between an industrial investment and a financial investment.

An entrepreneur who sets up his own company does not act like a financial investor, as he owns just one investment. As his assets are not diversified, it is a matter of "life or death" for the firm that the investment succeeds. The law of averages in risk diversification does not apply to him.[1]

The financial investor, on the other hand, needs portfolio management tools to estimate the risk–return on each of his investments. Portfolio theory is not the main objective here, but it is useful to introduce some basic notions with which financial managers must be familiar.

1 *However, the very fact that he does not diversify his portfolio means that he must achieve strong performances in managing the company, as he has everything to lose. So he's likely to take steps to reduce risk.*

Section 22.1
RETURN REQUIRED BY INVESTORS: THE CAPM[2]

The CAPM (Capital Asset Pricing Model) was developed in the late 1950s and 1960s. Based on the work of Harry Markowitz, William Sharpe, John Lintner and Jack Treynor, it is now universally applied.

The CAPM is based on the assumption that investors act rationally and have at their disposal all relevant information on financial securities (see "efficient markets" in Chapter 21). Like the investor in Chapter 27, they seek to maximise their return, at a given level of risk.

From portfolio selection, we know that the investor:

1 considers the efficient portfolios, that is, the portfolios that offer the highest return for a given level of risk (measured by the standard deviation);
2 introduces the risk-free asset. The tangent point between the risk-free asset and the efficient frontier is the portfolio with the highest ratio of risk premium to standard deviation; and
3 holds the same portfolio as everybody else as long as there are **homogeneous expectations** among investors. This portfolio is the **market portfolio**.

From the analysis of risk of individual securities, it has been shown that:

1 the contribution of a stock to a portfolio depends on the stock's sensitivity to the returns of the portfolio; and
2 the sensitivity to the returns of the market portfolio is known as beta (β).

2 *See also Appendix 22.A for a formal derivation of the CAPM.*

The CAPM says that if all investors hold the market portfolio, the risk premium they will demand is proportional to market beta.

The expected return of an asset will then be a linear function of beta:

$$\text{Expected return on a financial asset} = \text{Risk-free rate} + \beta \times \left(\text{Expected return of market portfolio} - \text{Risk-free rate}\right)$$

Remember that in order to minimise total risk, investors seek to reduce that component which can be reduced, i.e. the specific risk. They do so by diversifying their portfolios.

As a result, when stocks are fairly valued, investors will receive a return only on the portion of risk that they cannot eliminate – the market risk, or the nondiversifiable risk. Indeed, in a market in which arbitrage is theoretically possible, they will not be amply remunerated for a risk that they could otherwise eliminate themselves by simply diversifying their portfolios.

Portfolio theory's essential contribution is to show that an investor's required rate of return is not linked to total risk, but solely to market risk. Conversely, in a fairly valued market, intrinsic, or diversifiable, risk is not remunerated.

This means that the required rate of return (k) is equal to the risk-free rate r_F,[3] plus the risk premium for the nondiversifiable risk, i.e. the market risk.

This can be expressed as follows:

Required rate of return = risk-free rate + $\beta \times$ market risk premium, or:

$$k = r_F + \beta \times (k_M - r_F)$$

Where k_M is the required rate of return for the market and β the sensitivity coefficient described previously.

Note that the coefficient β measures the nondiversifiable risk of an asset and not its total risk. So it is possible to have a stock that is, on the whole, highly risky but with a low β if it is only loosely correlated with the market.

The difference between the return expected on the market as a whole and the risk-free rate is called the **equity risk premium**. This averages 3–5% in developed economies, but is higher in emerging markets.

The pattern of excess returns for equity holdings is observed in every country with a significant capital market. The US, along with the UK, Japan, Germany and France, account for more than 85 percent of the capitalised global equity value. According to the excellent retrospective of Mehra and Prescott (2003), the annual return on the British stock market was 5.7 percent over the post-war period, which is an impressive 4.6 percent premium over the average bond return of 1.1 percent. Similar statistical differentials are documented for France, Germany and Japan. The table below illustrates the equity premium in the post-war period for these countries.

3 *For the risk-free rate, k_F is equal to r_F. The required rate of return is equal to the return that is actually received, as the asset has no risk.*

Country	% real return on a market index	% real return on a relatively riskless security	% equity premium
UK (1947–1999)	5.7	1.1	4.6
Japan (1970–1999)	4.7	1.4	3.3
Germany (1978–1997)	9.8	3.2	6.6
France (1973–1998)	9.0	2.7	6.3

1/ THE EXPECTED RISK PREMIUM OVER TIME

MARKET RISK PREMIUM IN EUROPE

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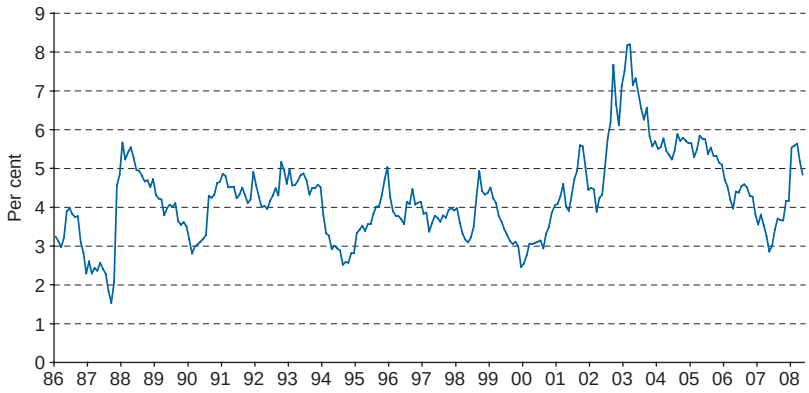

Source: Associés en Finance.

MARKET RISK PREMIUM IN THE USA

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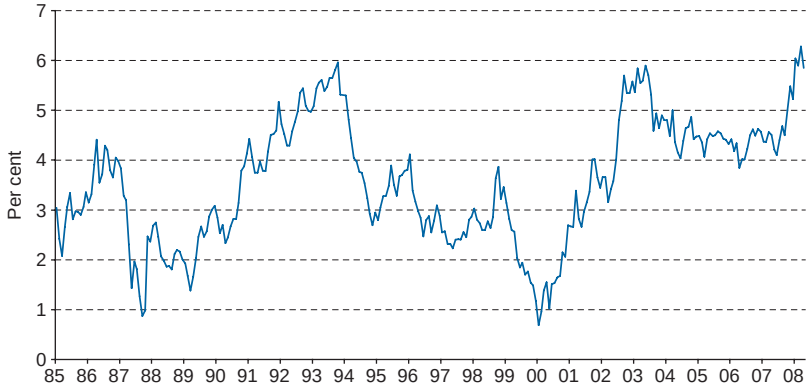

Source: BNP Paribas.

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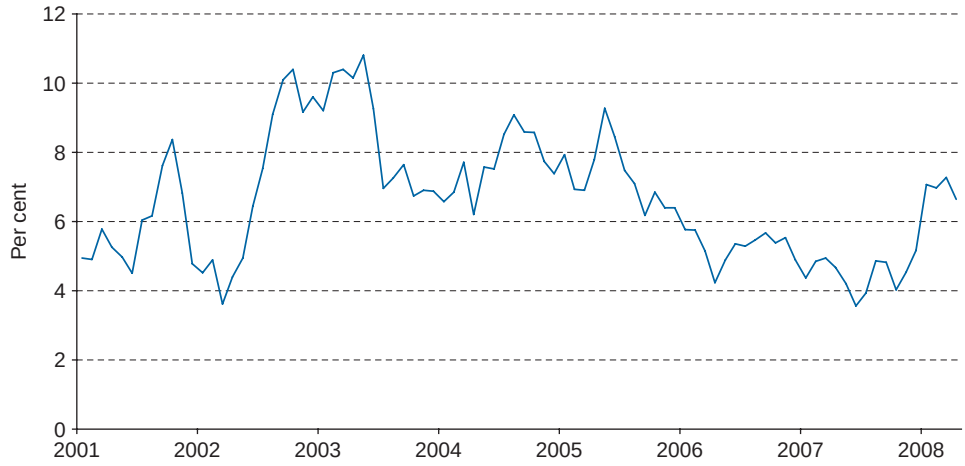
MARKET RISK PREMIUM IN EMERGING MARKETS



Source: Calyon.

Over the very long term (105 years!), the historical risk premium is as follows:

Germany (excluding 1922 and 1923)	5.3%
Belgium	2.6%
US	4.5%
France	3.9%
Italy	4.3%
Japan	5.9%
Netherlands	3.9%
United Kingdom	4.1%
Switzerland (since 1911)	1.8%

Source: Dimson et al. (2006).

Estimates of the equity market risk premium, i.e. the difference between the market return and the risk-free interest rate, are currently arrived at using two possible approaches:

- Either on the basis of forecast data (future free cash flows) and the current share price from which, after a few calculations, we deduct, the discount rate used, and thus the risk premium since the discount rate is equal for the whole of the market at the risk-free interest rate plus the risk premium. In this case, we refer to the **expected or**

forward risk premium, because it is based on investors' current expectations, both anticipated and not anticipated.

- Or on the basis of historical data relating to rates of returns received by investors over very long periods. In efficient markets, historical rates of return should be equal to future rates of return. In this case we refer to the **historical risk premium**, based on the very pleasing principle that, over the very long term, we end up getting what we ask for.

The historical risk premium is based on a comparison of annual performance of equity markets (including dividends) vs. the long-term risk-free rate. Expected risk premium is not directly observable. However, it can be calculated by estimating the future cash flows of all the companies, and then finding the discount rate that equates those cash flows with current share prices. It is this expected risk premium that is used in the CAPM.

To determine the risk premium for each stock, simply multiply the market risk premium by the stock's beta coefficient.

Hence, if the risk-free rate is 4.41% and the expected risk premium is 4.84%, the SAP shareholder will expect a return of $4.41\% + 1.73 \times 4.84\% = 12.78\%$, if SAP's β is 1.73, while the Carrefour shareholder will expect: $4.41\% + 0.73 \times 4.84\% = 7.94\%$, as Carrefour's β is 0.73.

A final remark. In the CAPM, the equilibrium risk premium is equal to the variance of the market portfolio times a weighted average of the degree of risk aversion of the holders of wealth (A):

$$E(r_M) - r_F = A \times \sigma_M^2$$

A should be considered as an index of the degree of risk aversion prevailing in the economy.

From a conceptual point of view, only the expected risk premium is acceptable for calculating a discount rate.

The price of an asset today can only correspond to expected discounted cash flows that it should generate given the rate of return required by the investor today. So, if the $/€ exchange rate is currently 1.53, it is at this price that it is possible to buy or sell the dollar – not at 1.20, even though this is the average exchange rate over the previous years. The historical premium has, de facto, three drawbacks:

1 Given the volatility of annual returns recorded (annual returns of + 20% or − 20% are not rare), calculations have to be based on data over a very long period in order to reduce the standard deviation of observations and to arrive at a relevant average. Even over 75 years, the theoretical standard deviation of observations following a normal rule is 2.5%, which means that a premium of 5%, for example, has as much chance of being 2.5% as 5% or 7.5%. So UBS estimates that the risk premium for the USA calculated by Ibbotson since 1926, often cited and used (7.1% on arithmetical average and 5.2% on geometric average as of 2007), would change by one point if it were calculated from 1925 or 1927.

2 When markets are rising, the historic rate of return achieved increases; thus the risk premium – calculated as an average including recent years in which performances were good – rises while, because the market is performing so well, rates of return required by shareholders may be falling. Similarly, when markets are falling

(2000–2003, 2007–2008), rates of return achieved are negative and bring down the historic average which takes them into account. At the same time, investors required returns rise.

3 Calculations of historical returns ignore the case of firms that went bankrupt over the period studied as this method only looks at the performance of share prices of firms still in existence today. However, the basis of a rate of return is the remuneration of the risk that a firm that goes bankrupt could generate only at a given moment. So it's hardly surprising that, using this method, we arrive at a higher risk premium (around 7%) than with the prospective method (just under 4% currently), as it ignores the case of investments with a −100% return (bankruptcy). It's a bit like including in a survey only those who have passed all of their A-levels to measure the average level of education of all 18-year-olds.

From a practical point of view, we believe that those using the historical approach to the risk premium, often driven by the fact that the current risk premium tends to be too volatile, forget that the risk-free interest rate can be even more volatile. Accordingly, proponents of the historical approach could, on the basis of the same argument of volatility, be justified in calculating the average risk-free interest rate over a long period. They don't do so because they probably realise the absurdity of the result based on a parameter, the risk-free interest rate, that is readily available in daily newspapers. In addition, it is inconsistent to calculate the risk premium using a given risk-free interest rate and in the formula of the CAPM, replacing *rF* with a different figure from that used to calculate the risk premium (because most of the time we don't know what risk-free interest rate was used to calculate the risk premium). Finally, those with many years' experience in calculating expected risk premiums know that when interest rates rise, the risk premium tends to fall. Finally, the required rate of return doesn't rise as high as the performance of the risk-free interest rate might lead one to assume, as the risk premium absorbs part of the rate hike (and vice versa in the event of a fall in interest rates). All things considered, the rate of return required by the shareholder, calculated using expected risk premia, is less volatile than the rate of return calculated using a constant risk premium.

As there are several available sources for the expected risk premium over the very short term (a few days to a few weeks), averages of these various sources can be calculated over a period of a few (for example, three) months. Beware of the risk that the use of averages calculated over long periods could be disconnected from the market.

Section 22.2
PROPERTIES OF THE CAPM

1/ THE SECURITY MARKET LINE

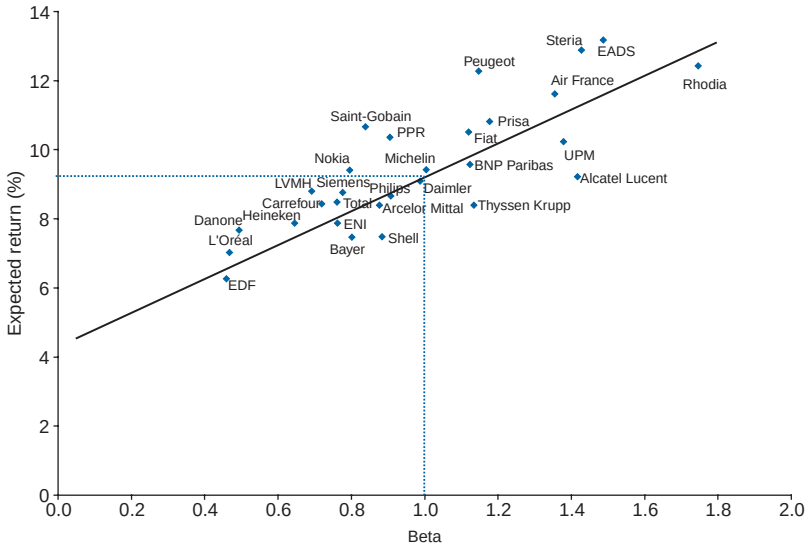
The French organisation Associés en Finance publishes the securities market line[4] for the entire euro zone. It is calculated on the basis of the **expected return** on the Y-axis and the **beta coefficient of each stock** on the X-axis.

Chapter 32 examines how the value of a stock can be expressed in terms of the flow of future dividends that it is expected to pay. These dividends are discounted at the rate

4 *It differs from the capital market line, which has the total risk σ of the security on the X-axis, not the β coef cient.*

required by the shareholder. The expected return is thus calculated by equating the current price of the stock to the discounted sum of dividends paid out to the company to infinity.

The securities market line is quite instructive. It helps determine the required rate of return on a security on the basis of the only risk that is remunerated, i.e. the market risk.



Example from early 2002.

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Source: Associés en Finance, 2008.

Shifts in the securities market line itself characterise the nature of changes in the markets and make it easier to understand them:

- a parallel shift, with no variation in slope (i.e. risk premium) reflects a change in the market brought on by a change in interest rates. For example, a cut in interest rates normally leads to a downward shift and thus a general appreciation of all stocks;
- a non-parallel shift (or pivoting) reflects a change in the risk premium and thus in the remuneration of risk. In this case, the riskiest stocks will move the most, whereas the least risky stocks may not be significantly affected.

In addition, the position of points *vis-à-vis* the market line serves as a decision-making tool. The above chart tells us that Peugeot offers too high an expected return for its risk. Investors will realise this and buy it, thus raising its price and lowering expected return. A stock that is "above" the securities market line is thus undervalued, while a stock that is "below" the securities market line (like Shell) is overvalued.

But do not rush to place an order. Since this chart was printed, prices have had plenty of time to adjust.

2/ LINEARITY

An important property of CAPM is that the measure of risk for individual assets is proportional to the weight of each security when the assets are combined into a portfolio.

Thus, for example, if an investor buys $a\%$ of asset X that has a systematic risk of β_X and $1 - X$ of the total wealth into asset Y with a systematic risk of β_Y, then the beta of the portfolio would simply be the weighted average of the betas of single securities.

This property is extremely useful if we want to compute the beta of a diversified company. We could in fact consider the beta of single operating businesses and then weight each of them according to their relative market capitalisation (alternatively, we could use sales or other dimensions).

Section 22.3
THE LIMITS OF THE CAPM MODEL

The CAPM model assumes that markets are efficient and it is without a doubt the most widely used model in modern finance. But financial analysts are always quick to criticise and thus this section appeases the critics by summarising how the CAPM presents some problems in practice.

1/ THE LIMITS OF DIVERSIFICATION

The CAPM model is a development of portfolio theory and is based on the assumption that diversification helps reducing risk (to the nondiversifiable risk). A study by Campbell et al. (2001) shows that diversification is increasingly complex and that, whereas in the 1970s a portfolio of 20 stocks reduced risk significantly, today at least 50 are required to achieve the same result.

This is due, among other things, to the greater volatility of individual stocks, although markets as a whole are no more volatile. Other reasons for this phenomenon are the arrival to market of riskier companies, such as biotechnology, Internet and younger companies, and the dwindling prominence of conglomerates which, by nature, provided some diversification in and of themselves.

Meanwhile, the correlation between market return and return on individual stocks is falling. This may undermine the relevancy of the CAPM model. Statistically, beta is becoming less and less relevant.

2/ DIFFICULTIES IN PRACTICAL APPLICATION OF THE CAPM

The first difficulty one encounters when using the CAPM is determining the risk-free rate which, all things considered, is just a theoretical concept.

The term “risk-free” means no risk of default and no coupon reinvestment risk. Zero-coupon government bonds come the closest to meeting this definition. However:

1 While governments’ power to mint currency means that their risk of default is low, they still have some risk (the Argentine default in 2002 is one illustration).
2 In order to have zero reinvestment risk on the coupon, the reference period must be known.

In the case of an asset paying a return only once in five years, it may be easy to use the zero coupon rate produced by the yield curve as a risk-free rate (see Chapter 24). However, the single-period CAPM model is often used to value assets whose cash flows are spread out

over time. So, theoretically, we would have to use a different discounting rate for each of the periods. Each of these rates would have to be calculated with a different risk-free rate, thus complicating the use of the CAPM model. In practice, this is done only to value bonds (see Chapter 25). For other assets, the risk-free rate at best reflects no more than the asset's duration.

Richard Roll has pointed out that determining a market portfolio is not as easy as one would like to think. In theory, the market portfolio is not made up solely of stocks nor even just financial assets, but of all the assets that can be acquired. It is therefore impossible, in practice, to come up with a true market portfolio, especially when looking at it from an international point of view.

However, this is not an insurmountable obstacle. Indeed, in a portfolio already containing a large number of assets, the marginal contribution to return of a new asset is low. Portfolio diversification makes return and risk approach a limit — the return offered by a theoretically ideal market portfolio. So the market portfolio can be approximated with a portfolio containing "only" a large number of assets. Unfortunately, recent studies have shown that more and more assets must be included in portfolio for it to be considered highly diversified.

However, we would still have to determine the return expected from the market portfolio. As the CAPM model is used for making forecasts, it can also be used to calculate the return expected from a security based upon the return expected from the market portfolio, as well as the security's anticipated risk (its β). However, "anticipated" data cannot be observed directly in the market, and so forecasts must be done on the basis of historical data and macroeconomic data. For some countries, such as emerging nations, this is not easy!

3/ THE FORECAST β

The main criticism of beta is its instability over time. It boils down a large amount of information into a single figure, and this strength becomes its weakness.

The CAPM model is used to make forecasts. It can be used to calculate expected return on the basis of anticipated risk. Therefore, it would be better to use a forecast β rather than an historical value, especially when the coefficient is not stable over time.

For this reason, calculations must often be adjusted to reflect the regularity of earnings and dividends, and visibility on the sector. Blume (1975) has sought to demonstrate a convergence of β towards 1 over the long term and has thus suggested the following adjustment to the standard equity β:

$$\beta_{Adjusted} = \beta_{Historic} \times \frac{2}{3} + \beta_{Market} \times \frac{1}{3}$$

Knowing that β_{Market} is 1, the value of β we get with the Blume adjustment is always closer to 1 than the historic β.

4/ RISK PREMIUM ESTIMATE AND TIME DIVERSIFICATION

To many, the size of the equity premium is a mystery. The figures that have been calculated seem to imply that investors historically required a high degree of compensation to invest in stocks rather than government bonds. Financial economists refer to the surprisingly high equity premium in the United States as the **equity premium puzzle**.

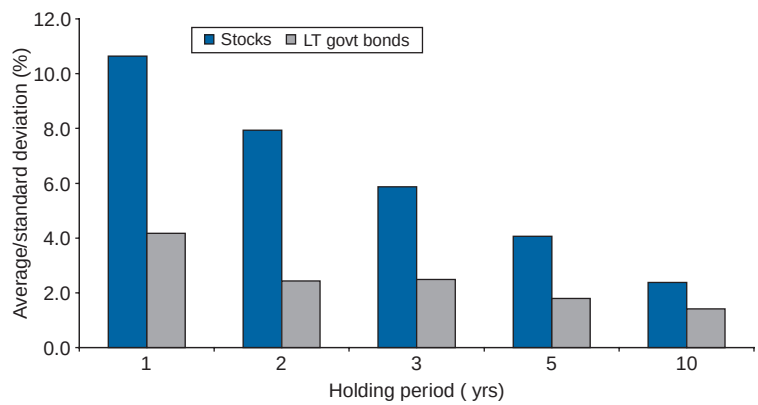
Equity premiums are a function of the volatility of the economy and of the risk associated with that particular market.[5] Keeping this in mind, here are a few hints for the reader who wants to estimate his own "home-made" equity premium:

5 *Thus we would expect riskier markets to have larger premiums than other nancial markets that are perceived as less risky.*

1 The risk premium should be measured over the longest period possible. Since stock returns are volatile, it is possible to obtain premiums with large standard errors if only short-time periods are used.
2 The type of average to be used is of importance. Assuming the returns being averaged are largely independent and that the future is like the past, arithmetic average is the best estimate of expected returns over a given future holding period. On the other hand, if our goal is to estimate returns over the next 10 or 30 years and returns are not independent, then geometric average is a better choice.
3 The equity risk premium can be measured by using any treasury security; however, in practice, only two securities are used. The first is short-term government bonds, which are sometimes preferred because they are the closest thing there is to a risk-free security. The second is the long-term government bond rates, chosen by those whose time horizon in financial analysis tends to be long term. The LT government bond raises some questions because, although free from default, they are not entirely risk-free securities.
4 The possibility of reducing the historical rate of return downward by $1\frac{1}{2}$ to 2% as a consequence of the **survivorship bias**. Brown et al. (1995) stated that survival imparts a bias to ex-post returns. A substantial upward bias would exist on markets that survive over a century without going bankrupt. Goetzmann and Jorion (1999) find that over a very long-period (1926–1996), the performance of the US market – during which time it did not experience an interruption in trading – exceeded the median return on a set of 11 countries with continuous histories by 1.9% in real terms or 1.4% in nominal terms.

The term "risk premium" implies that stocks are riskier than bonds. Is this conclusion always valid? Generally speaking, the differences in volatility between stocks and other investments depend on the holding period assumptions used to do the math. The risk premium is normally calculated using the annual total returns of financial securities. By extending the holding period (2 years, 5 years and so on), it is possible to see that standard deviation decreases substantially for stocks, and to a lower extent for bonds and other less volatile securities. This effect is called **time diversification**.

STANDARD DEVIATION OF RANDOM RETURNS FOR DIFFERENT HOLDING PERIODS



The phenomenon shown in the graph above was created with a simulation of random numbers for stocks and bonds. Assuming a 1-year holding period, the standard deviation of stocks is two to three times greater than the standard deviation of long-term government bonds. As the holding period increases, the standard deviation declines more rapidly for stock than for bonds. Using a 10-year holding-period return, the two assets have nearly identical riskiness. In conclusion, although stocks are surely riskier in the short run than bonds, in the long term this evidence tends to disappear.

Are long holding-period horizons relevant to investors? Yes, and much more so than investors realise. As Siegel (2002) highlights, "one of the greatest mistakes that investors make is to underestimate their holding period. This is so because many investors think about the holding periods of a particular stock, bond, or mutual fund. But the holding period that is relevant for portfolio allocation is the *length of time the investors hold any stocks or bonds*, no matter how many changes are made among the individual issues in their portfolio" (Siegel, 2002, p. 29).

4/ THE THEORETICAL LIMITS OF CAPM AND MARKETS AT FAIR VALUE

The CAPM assumes markets are fairly valued. But markets are not necessarily always at fair value. The fact that technical analysis has become so prominent on trading floors shows that market operators themselves have doubts about market efficiency (see Chapter 21).

Moreover, the theory of efficient markets in general, and the CAPM in particular, is based on the premise that market operators have rational expectations. To be applicable, the model must be accepted by everyone as being universally correct. The development of parallel theories shows that this is not necessarily the case.

The bias mentioned above has led the CAPM to be considered as just one theoretical explanation for the functioning of the financial markets. Other theories and methods have been developed, but they have not (yet?) achieved the attractiveness of the CAPM, due to the simplicity of its concepts.

Section 22.4
MULTIFACTOR MODELS

1/ THE ARBITRAGE PRICING THEORY (APT)

In some ways, the APT (Arbitrage Pricing Theory) model is an extended version of the CAPM model. CAPM assumes that the return on a security is a function of its market risk and therefore depends on a single factor: market prices. The APT model, as proposed by Stephen Ross, assumes that the risk premium is a function of several variables, not just one, i.e. macroeconomic variables ($V_1, V_2, \ldots, V_n$), as well as a company "noise".

So for security J:

$$r_J = a + b_1 \times r_{V1} + b_2 \times r_{V2} + \ldots + b_n \times r_{Vn} + \text{Company specific variable}$$

The model does not stipulate which V factors are to be used. Ross's original article uses the following factors, which are based on quantitative analyses:

- non-anticipated variations in inflation;
- non-anticipated variations in manufacturing output;
- non-anticipated variations in the risk premium;
- changes in the yield curve

The risk premium is then the sum of the risk premiums on each of the variables:

$$r_J - r_F = b_1 \times (r_{V1} - r_F) + b_2 \times (r_{V2} - r_F) + \ldots + b_n \times (r_{Vn} - r_F)$$

To use this model, we must first identify the relevant variables of a single security, the corresponding risk premiums, and then measure the security's sensitivity to these variables. For example, a chemical or paper company is more sensitive to overall economic growth than a maker of video games or a water distributor.

If all the *b* coefficients are zero, the risk premium is nil and the security's return is the risk-free rate.

In building a portfolio, an investor can adjust the various *b* values to obtain the desired level of risk. The most frequently used values are the difference between long-term and short-term government bond yields, changes in short-term interest rates, changes in effective exchange rates, changes in economic growth rates and non-anticipated changes in inflation.

Comparing the APT model to the market portfolio, we can see that APT has replaced the notion (hard to measure in practice) of return expected by the market with a series of variables that unfortunately must still be determined. This is why APT is a portfolio management tool and not a tool for valuing stocks.

2/ THE FAMA-FRENCH MODEL

There are offshoots from the APT that have sought to explain historical returns by company-specific factors rather than the general macroeconomic factors in the APT. For example, Eugene Fama and Kenneth French (1995) have isolated three factors: market return (as in the CAPM model), price/book value (see Chapter 32), and the gap in returns between large caps and small caps (which lends credence to the notion of a liquidity effect).

Other factors can be added to this list, including P/E, market capitalisation, yield and even past performance (which is a direct contradiction of efficient market theory). However, these are based on purely empirical approaches, not theoretical ones. While they criticise the CAPM model, they offer no better alternative model.

3/ LIQUIDITY PREMIUM, SIZE PREMIUM AND INVESTOR PROTECTION

Among factors used in determining risk, the criteria by which liquidity can be measured (size, free float, transaction volumes, bid–ask spread) are often statistically significant. In other words, the required return on a security often appears to be a function of liquidity.

In order to avoid confusion, it is preferable to separate the liquidity premium due to free float, transaction volumes, bid–ask spread from the so-called "size premium". Size premium is the additional remuneration due to the higher risk and, therefore, the higher cost of capital, associated with the idea of smaller size of the company and of the trading volume.

A number of studies have argued that the expected return and the cost of capital are inversely related to liquidity as represented by free float, transaction volume and bid–ask spread. If this is the case, an interesting consequence is that it is much easier to increase the liquidity of the firm's stock, while it can be quite difficult to lower the risk of a firm. Thus, it might be easier for a firm to lower its cost of capital through liquidity enhancement than change its risk profile.

What can corporations do in order to increase the liquidity of stocks? Mendelssohn and Amihud (2000) suggest two possible strategies:

1 they could try to bring in more uninformed investors. Stock splits may be useful in this regard; or
2 they could disclose more information.

Ibbotson Associates has broken down the NYSE stock returns into deciles by size, as measured by the aggregate market value of the common equity. The results show that the excess returns over the basic general equity risk premium increase with decreasing size, as shown in the table below.

Decile	Beta	Size premium (beta adjusted) (%)	Biggest company by market value ($m)
Mid-cap, 3rd–5th dec.	1.12	0.97	7777
Low-cap, 6th–8th dec.	1.22	1.76	1947
Micro-cap, 9th–10th dec.	1.36	3.88	627

Source: Morningstar–SBBI, Valuation Edition 2007 Yearbook.

If we expand CAPM to also reflect the size effect, we can expand the cost of equity capital formula to add this factor:

$$k = r_F + \beta \times (k_M - r_F) + \text{Size premium}$$

Hamon and Jacquillat (1999) have demonstrated the existence of a liquidity premium in Europe, which is nil for large caps and significant for small caps. The liquidity premium should be added to the return derived from the CAPM model to arrive at the total return expected by the shareholder. Hamon and Jacquillat use the term "market plane" (instead of securities market line). Under their model, expected return on a security is a linear equation with two parameters: the market premium and the liquidity premium. What is λ? Let us report the definition from the original article:

$$k = r_F + \beta \times (k_M - r_F) + \lambda \times \text{Liquidity premium}$$

In June 2008, Associés en Finance estimated the market plane parameters for euro-zone stocks at:

$$k = 1.87\% + \beta \times 5.68\% + \lambda \times 1.70\%$$

The liquidity premium, which is expected in addition to the required rate of return, finds its opposite number in the notion of "liquidity discount".

The liquidity premium was found in a study on the returns of several hundreds of European stocks.

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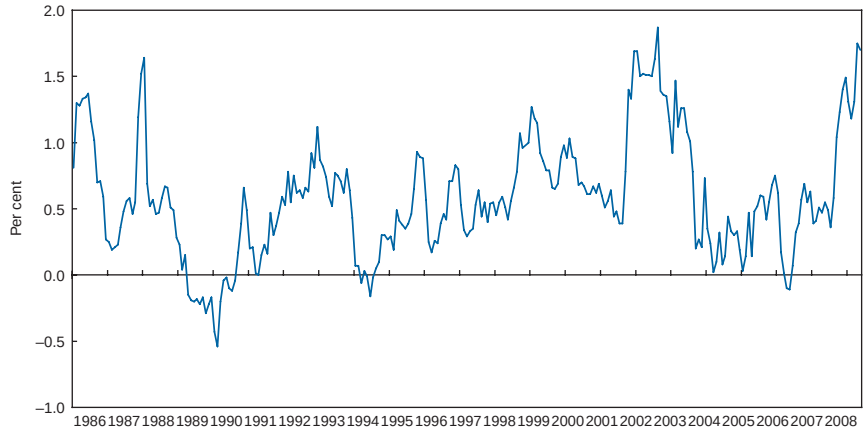
LIQUIDITY PREMIUM IN EUROPE



Source: Associés en Finance.

There is also some evidence that the cost of equity goes up where insider-trading laws are not enforced and legal protection of minorities is flawed. The legal system governing investors and markets in a given country *can influence systematic risk* because it determines the level of protection given to minority shareholders and other financial claimants.

LaPorta et al. (1999) report that companies in common law countries have higher valuations than companies in civil law countries. The median of medians of Tobin's Q is 1.23 for common law and 1.10 for civil law countries.

These results are consistent with the prediction that better shareholder protection is associated with higher corporate valuation. At the same time, the growth rate in sales is also higher for common law countries, suggesting that companies in those countries may face better investment opportunities.

Section 22.5
THE COST OF EQUITY BASED ON HISTORICAL RETURNS

The easiest and most intuitive way to estimate the cost of equity is to look at the past! If investors have rational expectations, i.e. if they think that on average they get what they expect, we may reasonably assume that the future returns will replicate past returns. Of course, this can only occur if the risk profile of the company in the past does not change significantly now or in the future.

Historical returns may refer to two types:

- market rate of returns; or
- accounting rate of returns.

1/ MARKET RATE OF RETURNS

The holding period return is the sum of two components: periodical dividends and the price appreciation over the single period. We may then assume that:

$$k_E \text{ is the average of the stream of } \frac{P_t - P_{t-1} + Div_t}{P_{t-1}}$$

Two comments:

1 length of the observation period. In principle, the longer the observed period, the higher the credibility of the average value. However, if we extend the length too much there is the risk that the company's risk profile will have changed in the meanwhile. Thus, the historical average may not entirely reflect the investors' expected rate of return.

2 nominal vs. real rate of return. It is usually better to estimate real rate of returns rather than the nominal ones. Inflation rates change each year and expected inflation may not coincide with the historical inflation rate. A solution to this problem is to use a two-step procedure to take inflation into account. First, the average real return is calculated. We can use the Fisher formula to obtain holding period return on a security net of inflation:

$$r_{real} = \frac{1 + r_{\text{nominal}}}{1 + \text{annual inflation}} - 1$$

where r indicates annual rate of return.

Given an estimate of the historical real return, the future long-run expected nominal return – our k_E – can be calculated by considering an estimate of long-run expected inflation:

$$k_E = (1 + \text{average } r_{real}) \times (1 + \text{expected inflation})$$

2/ ACCOUNTING RATE OF RETURNS

The fluctuations of market prices in inefficient markets may imply a low significance for the results obtained with the analysis of historical market returns. Or, historical market returns may be impossible to calculate for unlisted companies.

In these cases, we can use accounting rate of returns, which implies a two-step formula. First, we compute the after-tax ROI that we call k_E (unlevered):[6]

$$k_E\ (unlevered) = ROI \times (1 - T_C)$$

6 T_C = *corporate tax rate.*

Then we compute the "final" or levered k_E, using the target debt/equity ratio:

$$k_E = k_E\ (unlevered) + [k_E\ (unlevered) - k_D] \times (1 - T_C) \times \frac{D}{E}$$

However, we have some doubts about the validity of this methodology given that accounting data can be easily manipulated.

Section 22.6
THE COST OF EQUITY BASED ON CURRENT MARKET PRICES

The cost of equity can also be estimated by looking at current market prices. In this case, the analyst tries to extrapolate – through appropriate formulas – the cost of equity implicit in the stock prices. There are two alternative techniques that can be used to deduct the expected returns:

- the Dividend Discount Model (DDM); or
- the P/E model.

1/ THE DIVIDEND DISCOUNT MODEL (DDM)

The DDM is based on the assumption that the value of a company is given by the stream of dividends the investor expects to receive over a period of time. Assuming that:

- the growth rate of dividends and the cost of equity are constant from 0 to ∞; and
- the growth rate of dividends cannot exceed the cost of equity, then we can say – rewriting the Gordon formula (Chapter 27) – that the cost of equity is:

$$P_0 = \frac{Div_0 \times (1+g)}{k_E - g}$$

$$\text{i.e. } k_E = \frac{Div_o \times (1+g)}{P_0} + g$$

What if there are no dividends? In Chapter 38 it is shown that many companies around the world do not distribute dividends. One statistic provides valuable insight to help the reader understand the importance of this fact: in 2007, 77% of the S&P 500 constituents paid cash dividends, but only 39% of non-S&P 500 companies did. Fama and French (2001) report that the percentage of firms paying cash dividends fell from 66.5% in 1978 to 20.8% in 1999.

However, if the company pays dividends, there are two basic methodologies for estimating the growth rate, or *g*:

1 The historical growth rate of dividends. To apply this methodology a time series of dividends is necessary. The growth rate of dividends is calculated from period to period to determine the *average*.[7] This number is the growth rate of dividends.

7 *The reader has two alternatives: the* arithmetic *and the* geometric *average. Some textbooks suggest the arithmetic average (Brealey and Myers 2000, Pratt 2003) while others the geometric (Copeland et al., 2000). Consider the following example: you invest €1000 in a stock whose annual returns are +50% in t_1 and −50% in t_2. At the end of the first year the price of the share will be €1500; at the end of the second €750. The arithmetic average is 0; the geometric average is −13.4%. Mathematically, the geometric average is always lower than the arithmetic (unless all observations are equal) and the difference is a function of the volatility of returns. If we want to measure the change in value of our investment by looking at the past, we should prefer the geometric average. Conversely, if we want a standardised measure of performance, we should choose the arithmetic average. The arithmetic average assumes that returns are independent while geometric average assumes serial correlation among successive periods.*

2 The "internal" growth rate formula. According to this methodology, the growth of dividends is given by reinvesting cash flows in new projects, according to the following formula:[8]

$$g = ROE \times (1 - Payout)$$

8 *This formula assumes that the D/E ratio is kept constant. Thus, the company will increase its debt in proportion to reinvested earnings.*

2/ THE P/E MODEL

The Gordon formula can be rearranged in order to express the cost of capital as a function of the P/E ratio:

$$k_E = \frac{Div_o \times (1+g)}{P_0} + g = \frac{payout \times EPS_0 \times (1+g)}{P_O} + g = \frac{payout \times (1+g)}{P_O/EPS_O} + g$$

The assumptions are the same as for the DDM model. The growth rate of earnings can be estimated with the historical growth rate we described for the DDM. The P/E ratio should be the "trailing ratio".

The main difference between the DDM and the P/E model is that the former is based on the growth of dividends and the latter on the growth of earnings.

Section 22.7
MORE ON THE SHAREHOLDERS' REQUIRED RATE OF RETURNS

Shareholders cannot demand more from their investments than is warranted by the degree of risk. We refer here to the mythical figure of 15%, which is frequently proclaimed to be the minimum return required. In our view, this is a pipedream.

Under current market conditions, a return of about 10% on a share with average risk seems reasonable. This corresponds to the 10-year government bond rate of about 4.5%, plus a risk premium of 5.5%. Expecting 15% is too demanding and completely unjustified, unless the company has a beta of at least 1.9. No company belonging to the Eurostock 50 index falls into this category!

How can a company achieve these returns of 15% **in the long term**?

- **Debt leverage**. Using debt, the company can have a book return on equity that is higher than its return on capital employed. This can only be achieved if the return on capital is higher than its net debt charge. For a company with average risk to achieve a return on equity of 15%, its debt would probably have to represent 200% of equity. This is pretty much the case of France Telecom, but most companies do not have the luxury of its recurring cash flows! Fundamentally, leveraging debt to achieve 15% ROE significantly increases the risk for both investors and the company. This is normal behaviour during LBOs but, in those circumstances, all parties are aware of the risks incurred.

  Is this really what investors demanding 15% returns want?
- **Increasing operational risks**. To take an exaggerated example, a company could decide to expand in risky developing markets. In these nations, the normal returns are much higher than in the Euro area because the risks are much higher as well.

Is this really what investors demanding 15% returns want?

- **Creative accounting**. Two ways of accomplishing this are by employing the *pooling of interests* technique and using substantial asset write-downs. These two stratagems have roughly the same effect and both eliminate vast swathes of equity, thereby making it easier to achieve 15% book returns. These practices were used massively by groups like Vodafone and Vivendi. Obviously, these are just accounting tricks, and neither will last much longer in Europe. Shareholders will always demand their share of the returns on equity, regardless of whether some legal sleight of hand has eliminated some of the shareholders' equity from the balance sheet.

  Is this really what investors demanding 15% returns want?

- **Off-balance sheet financing**.[9] This is achieved when a company spins off its unprofitable assets to an unconsolidated entity, thus apparently improving its profitability. In fact, this is just another way of distorting reality and misleading the financial community.

  Is this really what investors demanding 15% returns want?

9 *More dif cult to perform under the IAS/IFRS principles.*

In the first two cases, the managers increase the risk for shareholders, who may thus expect to earn a 15% book or discounted return. But the price is that risk will increase accordingly. In the last two cases, corporate managers play with appearances and manipulate accounting rates of return. As is only logical, the impact on discounted rates of return is nil, but there will always be blind or naive investors who fall for these ploys.

In fact, as the preceding chapter discussed, economic theory in the medium term (and plain common sense!) tells us that the return corresponds to the normal rate of return required, given the risk incurred. This is reinforced by deregulation and technological advances, which have eroded the barriers to entry protecting economic rents. Many large groups like Saint-Gobain, Michelin, Air Liquide and even Coca Cola, all worldwide market leaders in their sectors, with patents, big name brands and powerful distribution networks in mature sectors, can either barely manage to cover, or earn just slightly more than, their cost of capital.

Today, companies create value when their investments yield 7–9% returns at average risk levels, and this is clearly well below the "magical" 15%. To demand 15% is to miss many investment opportunities that could create value.

In Europe, 15% returns at average risk levels are both unsustainable and unwarranted from a theoretical point of view. Practically speaking, such expectations on the part of the investor reflect wishful thinking. This type of attitude can lead to dangerous behaviour that encourages excessive debt levels and aggressive off-balance sheet accounting. It is hoped that managers aiming for 15% do so only after having deducted exceptional write-downs or goodwill from their capital. In reality, that would yield an overall return of 9% on the total equity contributed by shareholders. Under these circumstances, such behaviour and results would then be quite pardonable.

Shareholders can only expect returns compatible with the risks incurred. If they earn more, good for them, but this does mean they can systematically require more.

SUMMARY

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This chapter has shown how to work out the cost of equity, i.e. the rate of return required on equity capital. The investor's required rate of return is not linked to total risk, but solely to market risk. Conversely, in a market in equilibrium, intrinsic – or diversifiable – risk is not remunerated.

The CAPM (Capital Asset Pricing Model) is used to determine the rate of return required by an investor.

Risk-free rate $+ \beta \times$ market risk premium, or:

$$k = r_F + \beta \times (k_M - r_F) .$$

Although the CAPM is used universally, it does have drawbacks that are either practical (for reliable determination of beta coefficients) or fundamental in nature (since it supposes that markets are in equilibrium). This criticism has led to the development of new models, such as the Arbitrage Pricing Theory (APT), and has highlighted the importance of the liquidity premium for groups with small free floats. Like the CAPM, the APT assumes that the required rate of return no longer depends on a single market rate; however, it considers a number of other variables too, such as the difference between government bonds and Treasury bills, unanticipated changes in the growth rate of the economy or the rate of inflation, etc.

Two more techniques for estimating the cost of equity were discussed:

- the historical return method, where the cost of equity equals either the historical total market return or the accounting rate of return; and
- the current market price method, where the cost of equity can be extrapolated from current stock prices through appropriate formulae.

QUESTIONS

@ *quiz*

1/ Explain in a few lines why diversifiable risk cannot be remunerated on markets in equilibrium?

2/ Given that diversifiable risk is not remunerated, would it be worthwhile to diversify an investment?

3/ What is the rate of return required by the shareholder equal to?

4/ What is the drawback of the β coefficient?

5/ A shareholder requires a rate of return that is twice as high on a share with a β coefficient that is twice as high as that of another share. True or false?

6/ What does a low-risk premium indicate?

7/ On the graph on page 427, does the Alcatel-Lucent share seem under- or overvalued to you? What about the Daimler share?

8/ What is the strong point of the APT compared with the CAPM? And the weak point?

9/ Will liquidity premiums tend to rise or fall during a crash? Why?

10/ What does a negative liquidity premium indicate?

11/ The standard deviation of the earnings on Bouygues shares is 40%, while for Siemens it is only 28%. However, Bouygues has a β of 1.13 and Siemens of 1.7. Explain how this is possible.

12/ Explain why an investor would be prepared to require a lower return on a risk-free share for a share with a negative β.

13/ How do you explain the fact that rates of return required by investors may be identical for two groups of totally different activities (oil and IT services for example) as long as they have the same β.

14/ An experiment was recently carried out where a child, an astrologer and a financial analyst were each given €10,000 to invest for eight years. Who do you think achieved the best results?

EXERCISES

1/ What rate of return should be required on the Aventis share, which has a β of 0.7, if the Pinault Printemps Redoute share, which has a of β 1.1, returns 10% and is correctly valued, and the rate of a risk-free asset is 5%?

2/ Are the following shares undervalued, correctly valued or overvalued? The rate for a risk-free asset is 5.5% and the market risk premium is 4%.

Share	Air Liquide	Carrefour	Volkswagen	ING	Alcatel–Lucent
β	0.34	0.77	0.93	1.47	2.1
Rate of return	9%	8.2%	8%	10%	18%

3/ You think that the Alboni.com share will be worth €40 in one year. What price would you be prepared to pay today if the no-risk cash rate is 5%, the market rate of return is 9% and the β is 2.7?

4/ Your portfolio has a β of 1.2, the no-risk cash rate is 5.6% and the risk premium is 3%. In this chapter you learned about the APT and were told that the two *V* factors are growth of GDP and unanticipated inflation. The equation for the model is: $r_j = 5.6\% + b_{j1} \times 2\% + b_{j2} \times 5\%$. Suppose that the sensitivity of your portfolio to GDP growth is −0.4, what is your portfolio's sensitivity to unanticipated inflation? You believe that a recession is looming and you wish to eliminate your portfolio's sensitivity to GDP growth but you still want to get the returns you expected. What happens to your portfolio's sensitivity to unanticipated inflation?

5/ The Treasury bill rate is 5% and the market portfolio return is expected to be 13%.

(a) What is the market risk premium?
(b) What is the required rate of return on an investment which has a beta of 1.6?

(c) If the expected return on Pippus corporation is 17%, what is its beta?
(d) If an investment with a beta of 1.8% was expected to give a return of 19%, would it be convenient?

6/ What would the slope of the CML be if the degree of risk aversion increased from 2 to 3?

ANSWERS

Questions

1/ *Because if it were remunerated, this would be an "unwarranted" gain.*
2/ *Yes, in order to eliminate it, given that it is not remunerated.*
3/ *Risk-free rate + market risk premium.*
4/ *Its instability.*
5/ *No, because this would be forgetting the constant (the no-risk cash rate) in the equation for the required rate of return.*
6/ *That the market may be about to take a steep dive because risk is not being adequately rewarded.*
7/ *Overvalued, because the required rate of return, given the risk, is too low. It will thus rise, causing the share price to fall. Daimler is on the "securities market line" and is therefore correctly valued.*
8/ *Analysis of the market return in different components. The degree of precision required, because risk premiums by factor and the associated betas are difficult to estimate.*
9/ *To rise, because investors will only wish to invest in very liquid shares that they can sell immediately.*
10/ *A good thing for small companies, generally growing rapidly, which are in fashion at the time.*
11/ *The standard deviation is explained both by the market risk and the specific risk of the share, while the β only reflects the market risk of the share. Bouygues thus has a very high specific risk.*
12/ *These types of shares are very rare and very valuable, because they go up when the market falls! Their marginal contribution to the reduction of a portfolio's risk is thus strong.*
13/ *Because what is important in the CAPM is not the specific risk but the market risk of each security.*
14/ *The child. If markets are really efficient, the answer is completely random.*

Exercises

1/ *Risk premium: $(10\% - 5\%)/1.1 = 4.54\%$. $k = 5\% + 0.7 \times 4.54\% = 8.2\%$.*
2/ *Undervalued: Air Liquide, Alcatel-Lucent. Correctly valued: Carrefour Overvalued: Volkswagen, ING.*
3/ *$40\,€ / (5\% + 2.7 \times (9\% - 5\%) + 1) = 34.5\,€$.*
4/ *$r = 5.6\% + 1.2 \times 3\% = 9.2\%$. $(9.2\% - 5.6\% + 2\% \times 0.4)/5\% = 0.88$ $(9.2\% - 5.6\%)/5\% = 0.72$.*

5/ *(a) market risk premium is* $= 13 - 5 = 8\%$*;*
(b) required rate of return for an investment of beta of $1.6 = 5 + 1.6(13 - 5) = 17.8\%$
(c) return $= 5 + \beta(8) = 17, \beta = (17 - 5)/8 = 1.5$;
(d) required rate of return $= 5 + 1.8(8) = 19.4\%$*. The investment return is less than the required rate of return. Hence, its NPV will be negative.*

6/ *If the risk aversion increases from 2 to 3, the risk premium on the market portfolio would increase from .08 to .12, and the slope of the CML would increase from .4 to .6.*

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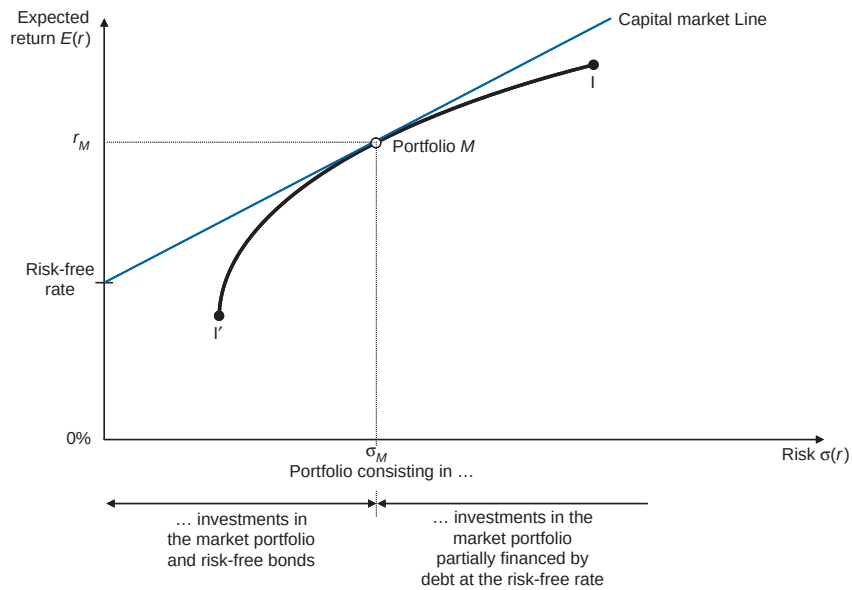
Appendix 22.A
A FORMAL DERIVATION OF THE CAPM

The figure below shows the expected return and standard deviation of the market portfolio, M, the risk-free asset, r_f, and a generic risky asset I. The line connecting the risk-free asset and the market portfolio is the *capital market line*.



We know that when markets are in equilibrium, the prices of all financial assets must adjust until there is no excess demand. Prices, in other words, must be fixed at a level where the supply of assets equals their demand. Therefore, the market portfolio for market in equilibrium includes all marketable assets held in proportion to their value weights.

The proportion of a single asset in the market portfolio is given by:

$$w_i = \frac{\text{Market value of a single asset}}{\text{Market value of all assets}}$$

Thus, a generic portfolio (P) consisting of a% invested in the single (risky) asset I and $(1 - a\%)$ in the market portfolio will have the following mean and standard deviation:

$$E(r_P) = aE(r_i) + (1 - a)\,E(r_M)$$
$$\sigma(r_P) = \left[a^2\sigma_i^2 + (1 - a)^2\,\sigma_M^2 + 2a(1 - a)\,\sigma_{iM}\right]^{1/2}$$

A crucial point is that the market portfolio already contains asset I held according to its market value weight because the market portfolio includes all assets held according to their market value weights.

The change in the mean and standard deviation with respect to the % of the portfolio, a, invested in asset I is determined as follow:

$$\frac{\partial E(r_P)}{\partial a} = E(r_i) - E(r_M)$$
$$\frac{\partial \sigma(r_P)}{\partial a} = \frac{1}{2}\left[a^2\sigma_i^2 + (1 - a)^2\,\sigma_M^2 + 2a\,(1 - a)\,\sigma_{im}\right]^{-1/2} \times \left[2a\sigma_i^2 - 2\sigma_M^2 + 2a\sigma_M^2 + 2\sigma_{iM} - 4a\sigma_{iM}\right]$$

Sharpe and Treynor's insight is that the *a%* is the excess demand for an individual risky asset. And we know that in equilibrium the excess demand for any asset must be zero. Therefore, if we evaluate the two equations above where excess demand equals zero, *we can then determine the equilibrium price at point M (i.e. the only point where there is no excess demand)*. We obtain:

$$\left.\frac{\partial E(r_P)}{\partial a}\right|_{a=0} = E\,(r_i) - E\,(r_M)$$
$$\left.\frac{\partial \sigma(r_P)}{\partial a}\right|_{a=0} = \frac{1}{2}\left(\sigma_M^2\right)^{-1/2}(-2\sigma_M^2 + 2\sigma_{iM}) = \frac{\sigma_{iM} - \sigma_M^2}{\sigma_M}$$

The slope of the risk-return trade-off evaluated at M is:

$$\left.\frac{\partial E(r_P)/\partial a}{\partial \sigma(r_P)/\partial a}\right|_{a=0} = \frac{E(r_i) - E(r_M)}{(\sigma_{iM} - \sigma_M^2)/\sigma_M}$$

The final step is to recognise that the slope of the opportunity set IMI' must also be equal to the slope of the capital market line $r_f M$. Since the slope of the capital market line is:

$$\frac{E(r_M) - r_f}{\sigma_M}$$

If we equate the slope of the risk-return tradeoff evaluated at M with the slope of the capital market line, we obtain:

$$\frac{E(r_M) - r_f}{\sigma_M} = \frac{E(r_i) - E(r_M)}{(\sigma_{iM} - \sigma_M^2)/\sigma_M}$$

Finally, we appropriately rearrange the equation by solving for $E(r_i)$:

$$E(r_i) = r_f + \left[E(r_M) - r_f\right] \frac{\sigma_{iM}}{\sigma_M^2}$$

This is the final equation of the **Capital Asset Pricing Model**: it states that the required rate of return on *any* asset is equal to the risk-free rate of return plus a risk premium. The latter is the *price* of the risk $[E(r_M) - r_f]$ multiplied by the quantity of risk (σ_{iM}/σ_M^2).

When shown graphically, this equation is also called the **security market line**.